

Problem Set 1 | Physics Review

1. **This must be done by hand** An athlete is working out using a device similar to the one below. The tension force

in each of the cables is 180 N. Assuming that the tension is applied on the right hand of the athlete at the point A , along the vector \vec{AC} , what is the moment of the force applied on point A at the athlete's right shoulder marked by B . The coordinates of A , B , and C are given as

$$A : (52, 126, 124) \text{ cm}$$

$$B : (27, 147, 99) \text{ cm}$$

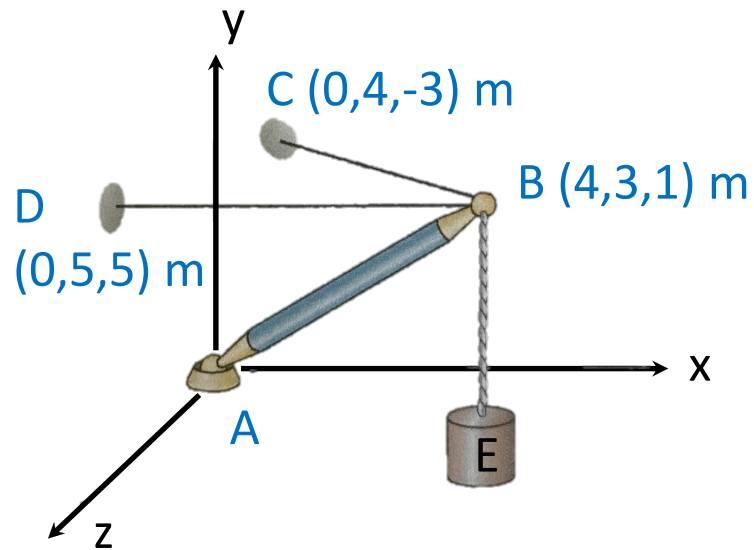
$$C : (32, 219, 188) \text{ cm}$$



Answer | $\vec{M}_B = -57.60\mathbf{i} - 32.97\mathbf{j} + 29.91\mathbf{k} \text{ Nm.}$

2. This must be answered using Matlab

Let us assume that a mass of E (100 kg) is hanging from point B . If the sum of moments at point A is zero, what are the magnitude of the tension forces in string BD

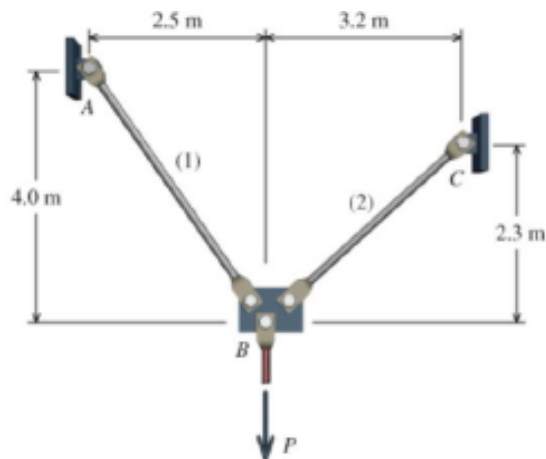


and BC ? For this problem assume that the mass of the bar AB is 20 kg and its weight is applied to the mid center of the bar. The mass of strings are negligible.

Answer | The magnitude of the tension forces are 555 N and 886 N

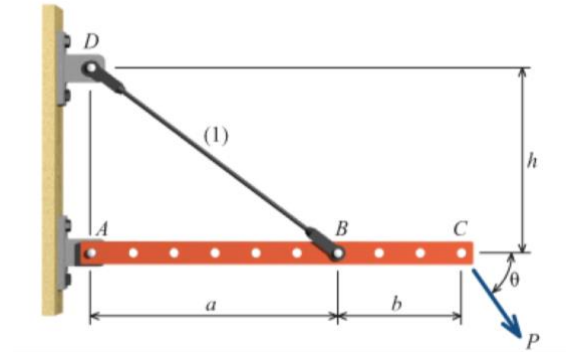
3. **This must be done by hand.** Two solid cylindrical rods support a load of $P = 20$ kN and the system is static. Calculate the forces in the beams.

Answer | $F_1 = \langle -8.63i + 13.8j \rangle$ kN $F_2 = \langle 8.63i + 6.2j \rangle$ kN



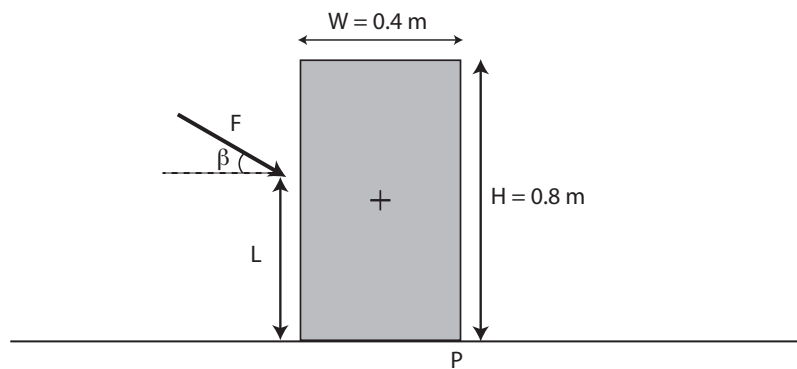
4. This must be completed with Matlab. The rod supports the rod (statically). The dimensions are $a = 600 \text{ mm}$, $b = 400 \text{ mm}$, $h = 400 \text{ mm}$, and $\theta = 60 \text{ degrees}$. There is a load $P = 1.2 \text{ kN}$. Find the reaction forces at A and the force magnitude in rod 1

Answer: $F_{ax} = 2.0 \text{ kN}$ $F_{ay} = -0.7 \text{ kN}$ $F_d = 3.1 \text{ kN}$



5. I have answered this question using ChatGPT. Try and answer the question yourself and compare with ChatGPT. GPT has made errors, point it out and reflect on how your work is similar/different from the AI.

A box of mass $M = 2 \text{ kg}$, width $W = 0.4 \text{ m}$, and height $H = 0.8 \text{ m}$ is resting on a rough surface with static and dynamic coefficients of friction $\mu_s = 0.4$. A force F is applied to the box at an angle $\beta = 15^\circ$ as shown.



- (a) Determine the minimum value of the force F that causes the box to slide.
- (b) Determine where the force F (as computed above) should be applied (L) to prevent the box from tipping. *Hint*: right before tipping the contact area between the box and the surface is reduced to point P.

Answer | (a) $F_{min} = 9.1 \text{ N}$; (b) $L = 0.55 \text{ m}$;



Let's solve the problem step-by-step.

Given:

- Mass of the box $M = 2 \text{ kg}$
- Width of the box $W = 0.4 \text{ m}$
- Height of the box $H = 0.8 \text{ m}$
- Static coefficient of friction $\mu_s = 0.4$
- Angle of applied force $\theta = 15^\circ$

Part (a): Determine the minimum value of the force F that causes the box to slide.

To cause the box to slide, the horizontal component of the applied force must overcome the maximum static friction force.

1. Calculate the normal force N :

$$N = Mg = 2 \times 9.8 = 19.6 \text{ N}$$

2. Maximum static friction force F_{friction} :

$$F_{\text{friction}} = \mu_s N = 0.4 \times 19.6 = 7.84 \text{ N}$$

3. Horizontal component of the applied force F :

$$F \cos \theta \geq F_{\text{friction}}$$

$$F \cos 15^\circ \geq 7.84$$

4. Solving for F :

$$F \geq \frac{7.84}{\cos 15^\circ} \approx \frac{7.84}{0.9659} \approx 8.12 \text{ N}$$

So, the minimum force F required to cause the box to slide is approximately 8.12 N.

Part (b): Determine where the force F should be applied (L) to prevent the box from tipping.

To prevent the box from tipping, the moment about the tipping point P should be considered. The tipping point is at the edge of the box's base.

1. Calculate the torque due to the applied force F :

The applied force F has a horizontal and vertical component:

$$F_{\text{horizontal}} = F \cos \theta$$

$$F_{\text{vertical}} = F \sin \theta$$

The torque τ_F about the tipping point P due to the applied force F at a height L from the base is:

$$\tau_F = F_{\text{horizontal}} \cdot H + F_{\text{vertical}} \cdot L$$

2. Calculate the torque due to the weight of the box:

The torque due to the weight of the box τ_{mg} about the tipping point P (assuming it acts at the center of gravity, halfway along the width $W/2$) is:

$$\tau_{mg} = Mg \cdot \frac{W}{2} = 2 \times 9.8 \times 0.2 = 3.92 \text{ Nm}$$

3. Setting up the equilibrium condition to prevent tipping:

For the box to be on the verge of tipping, the torque due to the applied force should be equal to the torque due to the weight:

$$\tau_F = \tau_{mg}$$

$$F_{\text{horizontal}} \cdot H + F_{\text{vertical}} \cdot L = 3.92$$

$$(8.12 \cos 15^\circ) \cdot 0.8 + (8.12 \sin 15^\circ) \cdot L = 3.92$$

$$(8.12 \cdot 0.9659) \cdot 0.8 + (8.12 \cdot 0.2588) \cdot L = 3.92$$

$$6.276 + 2.102L = 3.92$$

4. Solving for L :

$$2.102L = 3.92 - 6.276$$

$$2.102L = -2.356$$

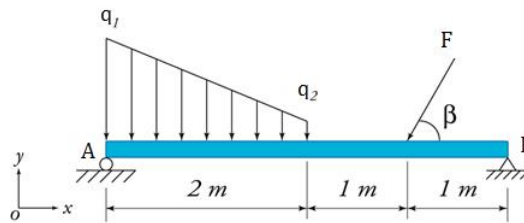
$$L = \frac{-2.356}{2.102} \approx -1.12 \text{ m}$$

Since the negative value for L indicates that the force cannot be applied in such a way to prevent tipping if the force is applied above the base of the box, the box will start to tip before it starts to slide if the force is applied at the height calculated. Therefore, the force should be applied as low as possible, ideally at the base (close to 0) to maximize the chances of sliding before tipping.

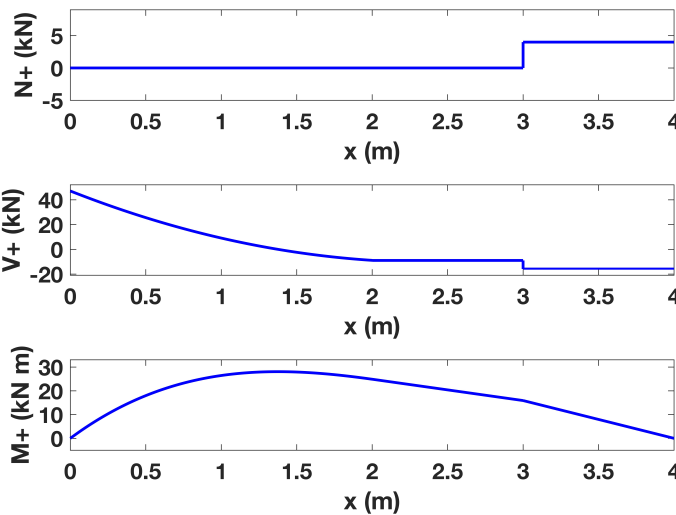
Problem Set 2:

Support Loads and Internal Loads (All Calculations in Matlab)

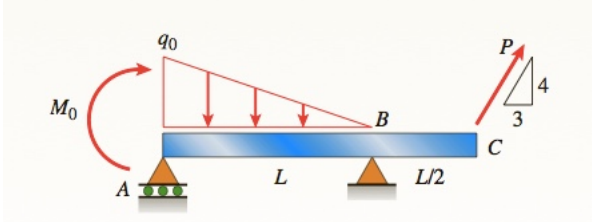
- Consider the uniform, horizontal beam shown in the figure. The beam is supported by a frictionless roller at A and hinged to the ground at B. The beam is subjected to the distributed and concentrated loads as shown in the figure. The concentrated force makes an angle $\beta = 60^\circ$ with the horizontal axis. The magnitude of the force is $F = 8\text{ kN}$. Given $q_1 = 48\text{ kN/m}$ and $q_2 = 8\text{ kN/m}$, determine the reactions at A and B for the beam. Calculate the internal axial force N , shear force V , and bending moment M at $x = 1\text{ m}$ from A. Show the diagrams of the internal loads. Report all calculations for full credit.



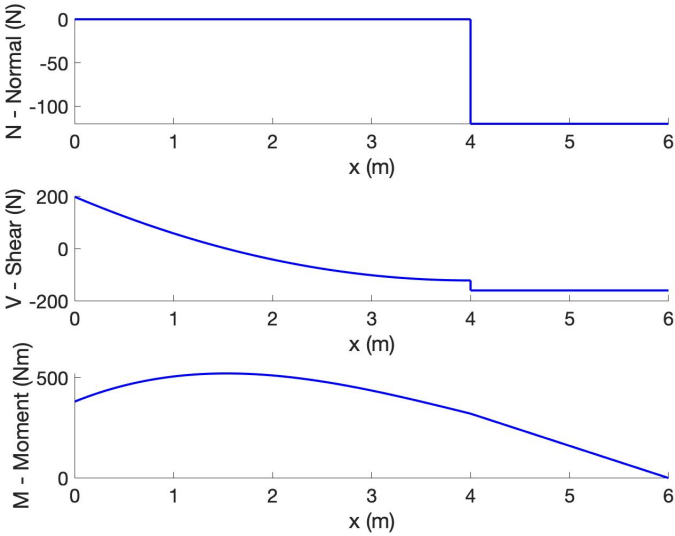
Answer | $A_y = 47.1\text{ kN}$; $B_x = 4\text{ kN}$; $B_y = 15.9\text{ kN}$; $N = 0\text{ kN}$; $V = 9.1\text{ kN}$; $M = 26.4\text{ kNm}$.



- Consider the uniform, horizontal beam shown in the figure. The beam is supported by a frictionless roller at A and hinged to the ground at B. Find support reactions at A and B. Calculate the axial force N , shear force V , and bending moment M at midspan of AB. Let $L = 4\text{ m}$, $q_0 = 160\text{ N/m}$, $P = 200\text{ N}$, and $M_0 = 380\text{ Nm}$. Show the diagrams of the internal loads. Report all calculations for full credit.



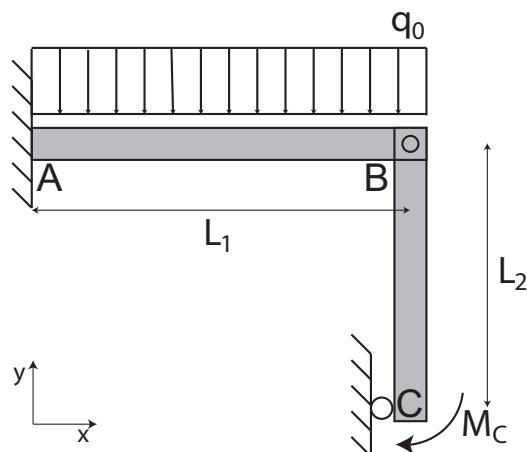
Answer | $A_y = 198.3N$; $B_x = -120N$; $B_y = -38.3N$; $N = 0N$; $V = -41.7N$; $M = 510Nm$.



3. I have answered this question using ChatGPT. Check the AI solution for errors and point them out and reflect on how your work is similar/different from the AI.

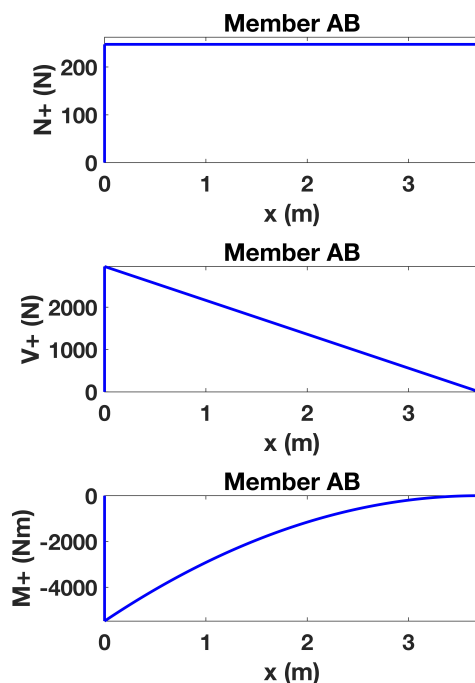
A plane frame is restrained with a **fixed support** at A and with a **roller** at C. Members

AB and BC are connected at B with an internal **pin**. A uniformly distributed load with intensity of $q_0 = 800\text{N/m}$ acts on AB. A concentrated moment $M_C = 680\text{Nm}$ is applied at C. $L_1 = 3.70\text{m}$, and $L_2 = 2.75\text{m}$.



- Find the reaction loads at supports A and C. *Hint:* Break the structure at B. Solve for equilibrium of member BC first, then solve for equilibrium of member AB.
- Calculate the axial force N , shear force V , and bending moment M at midspan of AB. Show the diagrams of the internal loads for Beam AB. Report all calculations for full credit.

Answer | (a) $A_x = -247.3\text{N}$; $A_y = 2960\text{N}$; $M_A = 5476\text{Nm}$; $C_x = 247.3\text{N}$; (b) $N = 247.3\text{N}$; $V = 1480\text{N}$; $M = -1369\text{Nm}$.



Step 1: Analyzing Member BC

Free Body Diagram of Member BC:

- C_x and C_y are the horizontal and vertical reaction forces at support C.
- B_x and B_y are the horizontal and vertical forces at joint B (internal pin).
- There is a concentrated moment $M_C = 680 \text{ Nm}$ applied at C.

Equilibrium Equations for BC:

1. Sum of forces in the x-direction:

$$\begin{aligned}\sum F_x = 0 &\implies C_x - B_x = 0 \\ &\implies C_x = B_x\end{aligned}$$

2. Sum of forces in the y-direction:

$$\begin{aligned}\sum F_y = 0 &\implies C_y - B_y = 0 \\ &\implies C_y = B_y\end{aligned}$$

3. Sum of moments about point B:

$$\begin{aligned}\sum M_B = 0 &\implies -M_C + C_y \cdot L_2 = 0 \\ &\implies 680 = C_y \cdot 2.75 \\ &\implies C_y = \frac{680}{2.75} \\ &\implies C_y = 247.27 \text{ N}\end{aligned}$$

Since $C_y = B_y$:

$$B_y = 247.27 \text{ N}$$

Horizontal Forces:

Since there are no horizontal loads on BC:

$$C_x = B_x = 0$$

Step 2: Analyzing Member AB

Free Body Diagram of Member AB:

- A_x and A_y are the horizontal and vertical reaction forces at support A.
- B_x and B_y are the horizontal and vertical forces at joint B (internal pin).
- There is a uniformly distributed load $q_0 = 800 \text{ N/m}$ acting on AB.

Equilibrium Equations for AB:

1. Sum of forces in the x-direction:

$$\begin{aligned}\sum F_x = 0 &\implies A_x - B_x = 0 \\ &\implies A_x = 0\end{aligned}$$

2. Sum of forces in the y-direction:

$$\begin{aligned}\sum F_y = 0 &\implies A_y + B_y - q_0 \cdot L_1 = 0 \\ &\implies A_y + 247.27 - 800 \cdot 3.7 = 0 \\ &\implies A_y + 247.27 - 2960 = 0 \\ &\implies A_y = 2960 - 247.27 \\ &\implies A_y = 2712.73 \text{ N}\end{aligned}$$

3. Sum of moments about point A:

$$\begin{aligned}\sum M_A = 0 &\implies -B_y \cdot L_1 + q_0 \cdot \frac{L_1^2}{2} = 0 \\ &\implies -247.27 \cdot 3.7 + 800 \cdot \frac{3.7^2}{2} = 0 \\ &\implies -247.27 \cdot 3.7 + 800 \cdot 6.845 = 0 \\ &\implies -914.10 + 5476 = 0 \\ &\implies 4561.9\end{aligned}$$

Step 3: Internal Forces at Midspan of AB

At Midspan of AB:

- $x = L_1/2 = 3.7/2 = 1.85 \text{ m}$

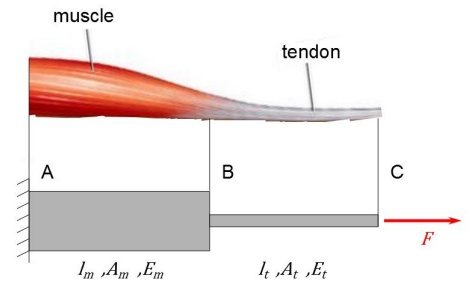
Axial Force, N :

Since there are no horizontal loads,

$$N = A_x = 0$$

Problem Set 3: Uniaxial Loading and Stress Transformation

1. Consider a muscle/ tendon complex as shown. A very crude two rod model can be used to approximate how the tendon and the muscle extend when the system is loaded with a force F . The muscle is modeled as a rod with length $l_m = 6\text{cm}$ and a diameter $d_m = 2.8\text{cm}$. The elastic modulus in this range can be assumed as $E_m = 60\text{kPa}$. The muscle is attached to the tendon at point B, which is modeled as a second rod with length $l_t = 3.8\text{cm}$ and a diameter $d_t = 0.7\text{cm}$.



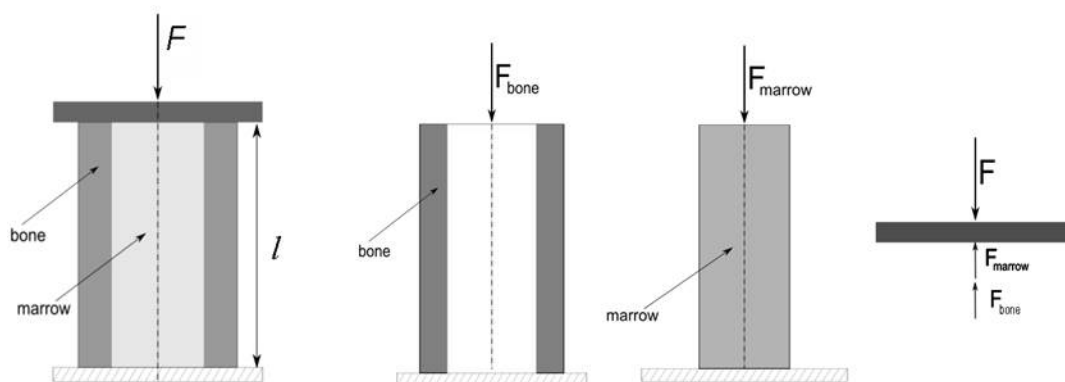
At point A the muscle is attached to the bone, which can be considered as a rigid fixation. At point C a force $F = 9\text{N}$ is applied parallel to the tendon which causes the overall length of the muscle/ tendon system to stretch to 12.2 cm.

- (a) Determine the modulus of the tendon E_t .
- (b) Determine the stresses σ_{AB} at a cross section between A and B and σ_{BC} a cross section between B and C.

Answer | (a) $E_t = 0.947\text{MPa}$; (b) $\sigma_{AB} = 0.0146\text{MPa}$; $\sigma_{BC} = 0.234\text{MPa}$.

2. A section of the mid femur shaft, itself of length L , can be assumed to be an outer tube (outer radius r_o , inner radius r_i) of bone (E_{bone}) and an inner cylinder of marrow (E_{marrow}). Assume this piece is loaded by a vertical force F , through a rigid plate, as shown. Determine the compressive stress (σ_{bone} , σ_{marrow}) and lengthening (δ_{bone} , δ_{marrow}) in the bone and in the marrow as a function of F .

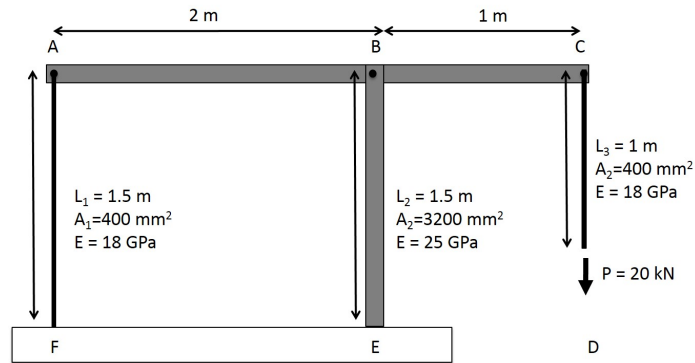
Answer $\sigma_{bone} = -\frac{FE_b}{E_m A_m + E_b A_b}$, $\sigma_{marrow} = -\frac{FE_m}{E_m A_m + E_b A_b}$ $\delta_{bone} = -\frac{FL}{E_m A_m + E_b A_b}$.



3. A rigid bar ABC is connected to two rods (AF and CD) and held up by a post (BE). The lengths, modulus, and area of the rods and post are shown. A downward force $P = 20\text{kN}$ is applied to rod CD. Determine:

the change in length of rods AF, CD, and post BE ($\delta_{AF}, \delta_{BE}, \delta_{CD}$)

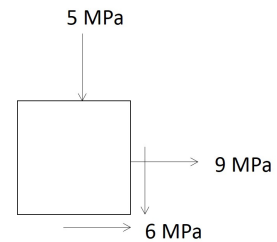
The rigid bar ABC does not bend or deflect but rotates about the internal pin B. *Hint:* Draw the free-body diagram for the bar ABC and for each rod separately.



Answer | $\delta_{AF} = 2.08\text{ mm}$, $\delta_{BE} = -0.56\text{ mm}$, $\delta_{CD} = 2.78\text{ mm}$

4. The state of stress is shown on the element. Determine:

- (a) the principal stresses (σ_1, σ_2) and the angle of principal stresses (θ_p)
- (b) the maximum shear stress (τ_{max}), the average normal stress at maximum shear (σ_{ave}), and the angle of maximum shear (θ_s).

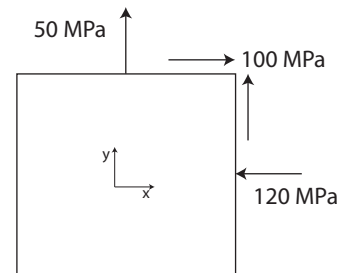


Draw the orientation of the element for both cases and the stresses in that orientation.

Answer | $\sigma_1 = 11.22 MPa, \sigma_2 = -7.22 MPa, \theta_p = -20.3^\circ$, b. $\tau_{max} = -9.22 MPa, \sigma_{ave} = 2 MPa, \theta_s = 24.7^\circ$

5. The state of stress is shown on the element. Determine:

- (a) the principal stresses (σ_1, σ_2) and the angle of principal stresses (θ_p)
- (b) the maximum shear stress (τ_{max}), the average normal stress at maximum shear (σ_{ave}), and the angle of maximum shear (θ_s).

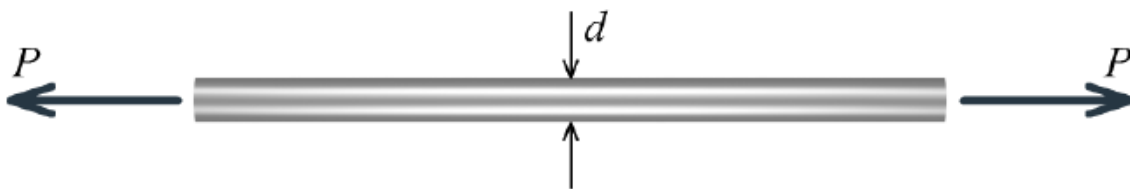


Draw the orientation of the element for both cases and the stresses in that orientation.

Answer | $\sigma_1 = 96.24 MPa, \sigma_2 = -166.24 MPa, \theta_p = -24.82^\circ$, b. $\tau_{max} = 131.24 MPa, \sigma_{ave} = -35 MPa, \theta_s = 20.18^\circ$.

6. A solid circular rod with a diameter of $d = 15 \text{ mm}$ is shown. The rod is made of an aluminum alloy that has an elastic modulus of $E = 70 \text{ GPa}$ and

Poisson's ratio of $\nu = 0.3$. When subjected to the axial load P , the diameter of the rod decreases by 0.23 mm . Determine the magnitude of the load P .

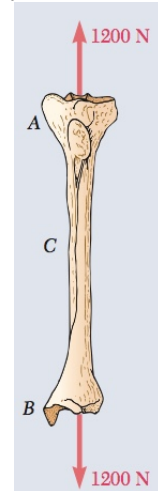


Answer: $P = 632 \text{ kN}$

7. I have answered this question using ChatGPT. Check the AI solution for errors and point them out and reflect on how your work is similar/different from the AI.

A mechanical testing machine is used to subject a bone

to two 1200N normal forces. A strain gage located at C on the surface of the bone indicates that the average normal stress in the bone is 3.80MPa . Assume that the cross section of the bone at C is a hollow circle with outer diameter $d_{outer} = 25\text{mm}$. Determine the inner diameter.



To determine the inner diameter of the bone given the provided information, we can use the relationship between stress, force, and area. Here's the step-by-step process:

1. Calculate the cross-sectional area (A) using the stress formula:

The average normal stress (σ) is given by the formula:

$$\sigma = \frac{F}{A}$$

where F is the force applied and A is the cross-sectional area.

Rearranging this formula to solve for A :

$$A = \frac{F}{\sigma}$$

2. Substitute the known values:

Given:

- Force $F = 1200\text{ N}$ (note: if there are two forces of 1200 N each, it could mean that the total force is 2400 N , but we'll proceed with each force as individual since they may be acting on different points.)
- Average normal stress $\sigma = 3.80\text{ MPa}$ (which is $3.80 \times 10^6\text{ Pa}$)
- Outer diameter $d_{outer} = 25\text{ mm}$

Let's convert the units of diameter to meters (since stress is in Pa):

$$d_{\text{outer}} = 25 \text{ mm} = 0.025 \text{ m}$$

3. Calculate the area (A):

$$A = \frac{1200 \text{ N}}{3.80 \times 10^6 \text{ Pa}} = 3.1579 \times 10^{-4} \text{ m}^2$$

4. Express the cross-sectional area of a hollow circle:

The cross-sectional area A of a hollow circle is given by:

$$A = \pi \left(\frac{d_{\text{outer}}^2}{4} - \frac{d_{\text{inner}}^2}{4} \right)$$

Rearranging to solve for d_{inner} :

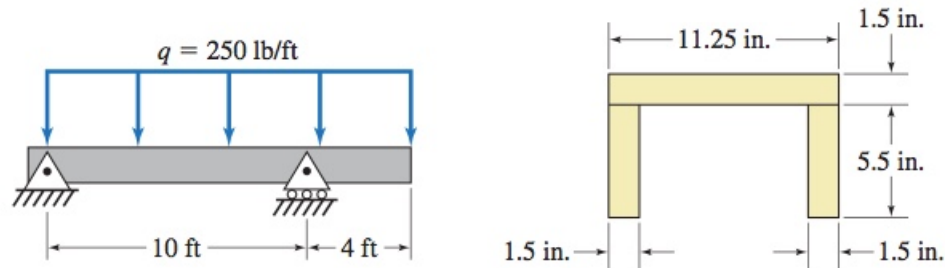
$$\frac{A}{\pi} = \frac{d_{\text{outer}}^2}{4} - \frac{d_{\text{inner}}^2}{4}$$

$$\frac{4A}{\pi} = d_{\text{outer}}^2 - d_{\text{inner}}^2$$

$$d_{\text{inner}}^2 = d_{\text{outer}}^2 - \frac{4A}{\pi}$$

Problem Set 4: Bending All Calculations in Matlab

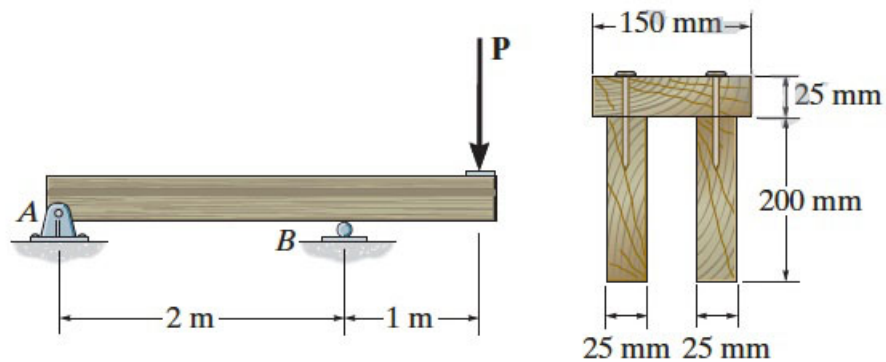
- The overhanging beam supported by a pin (left) and a roller (right) has a U-shaped cross-section. Determine the maximum tensile and the subsequent compressive bending stress in the beam.



Answer | $x = 4.2\text{ft}$; $\sigma_{max,tension} = 814\text{psi}$ and $\sigma_{compression} = 446\text{psi}$

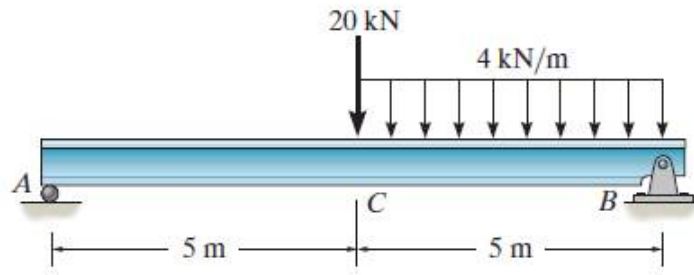
- The overhanging beam is made of wood with an allowable tensile stress of $\sigma_{tension} = 4\text{MPa}$

and an allowable compression stress of $\sigma_{compression} = -5\text{MPa}$. The beam is supported by a pin at A and a roller at B. Determine the maximum force P_{max} that can be applied at the end. *Hint: Calculate reaction forces in terms of P. Calculate the shear and moment diagrams in terms of P. Find the maximum/minimum moments in terms of P.*



Answer | Checks along the way: $\bar{y} = 130.68\text{mm}$, location of the neutral axis, measured from the bottom edge; $I = 68.04 \times 10^6\text{mm}^4$; $M_{min} = -P$; $P_{max} = 2.60\text{kN}$.

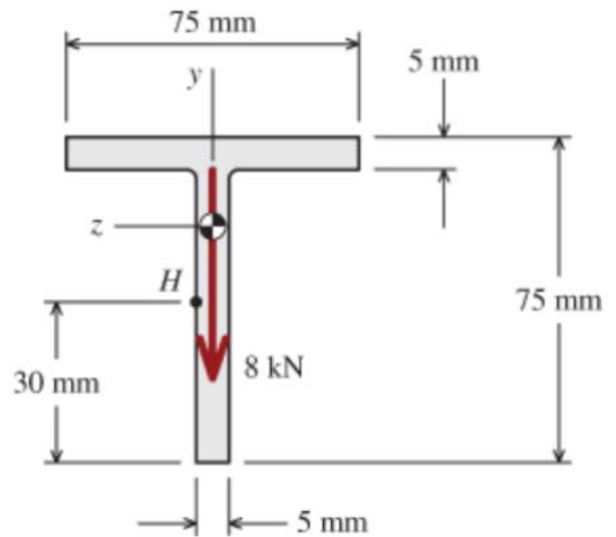
3. The beam is made of steel ($E = 200GPa$) and subjected to a point load and a distributed load as shown. Determine the deflection at its center C. $I = 0.1457 \times 10^{-3}m^4$.



Answer | $\Delta = -23 \text{ mm}$

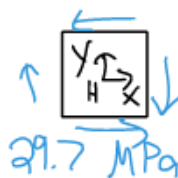
4. The internal shear force V at a certain section of an aluminum beam is 8 kN. If the beam has the cross section shown at right, determine:

- the shear stress at point H, which is located 30 mm above the bottom surface of the tee shape, and draw it on a stress element.
- the maximum horizontal shear stress in the tee shape and draw it on a stress element.



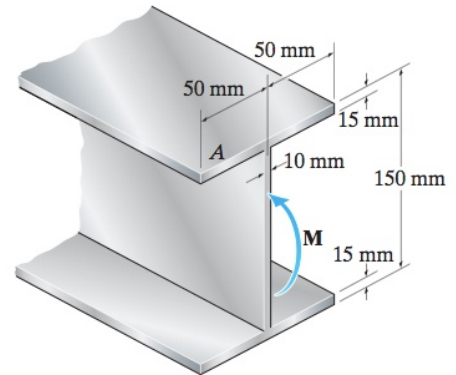
Answer | a) 23.7 MPa

b) 29.7 MPa



5. I have answered this question using ChatGPT. Check the AI solution for errors and point them out and reflect on how your work is similar/different from the AI.

Determine the maximum bending stress of an I-beam subjected to an internal moment of $M = 30kNm$.



To determine the maximum bending stress in the I-beam, we will use the flexure formula:

$$\sigma = \frac{Mc}{I}$$

where:

- σ is the maximum bending stress,
- M is the internal moment,
- c is the distance from the neutral axis to the outermost fiber,
- I is the moment of inertia of the cross-section.

First, we need to calculate the moment of inertia (I) for the I-beam cross-section about the neutral axis.

Given dimensions:

- Width of flanges: 50 mm
- Thickness of flanges: 15 mm
- Height of web: 150 mm
- Thickness of web: 10 mm

1. Calculate the area and centroid of the sections:

The I-beam consists of three rectangles (top flange, bottom flange, and web).

- Top flange:

- Area, $A_1 = b \cdot t_1 = 50 \text{ mm} \cdot 15 \text{ mm} = 750 \text{ mm}^2$
- Distance from the bottom of the top flange to the neutral axis:
 $y_1 = 150 \text{ mm} - 15 \text{ mm} - \frac{15 \text{ mm}}{2} = 127.5 \text{ mm}$

- Bottom flange:

- Area, $A_2 = b \cdot t_2 = 50 \text{ mm} \cdot 15 \text{ mm} = 750 \text{ mm}^2$
- Distance from the bottom of the bottom flange to the neutral axis:
 $y_2 = \frac{15 \text{ mm}}{2} = 7.5 \text{ mm}$

- Web:

- Area, $A_3 = t_w \cdot h = 10 \text{ mm} \cdot 120 \text{ mm} = 1200 \text{ mm}^2$
- Distance from the bottom of the web to the neutral axis:
 $y_3 = 75 \text{ mm}$

2. Calculate the neutral axis:

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{750 \cdot 127.5 + 750 \cdot 7.5 + 1200 \cdot 75}{750 + 750 + 1200}$$

Let's calculate the value:

$$\bar{y} = \frac{95625 + 5625 + 90000}{2700} = \frac{191250}{2700} \approx 70.83 \text{ mm}$$

3. Calculate the moment of inertia (I):

- Top flange:

$$I_1 = \frac{1}{12} b t_1^3 + A_1 (y_1 - \bar{y})^2 = \frac{1}{12} \cdot 50 \cdot 15^3 + 750 \cdot (127.5 - 70.83)^2$$

- Bottom flange:

$$I_2 = \frac{1}{12} b t_2^3 + A_2 (y_2 - \bar{y})^2 = \frac{1}{12} \cdot 50 \cdot 15^3 + 750 \cdot (70.83 - 7.5)^2$$

- Web:

$$I_3 = \frac{1}{12} t_w h^3 + A_3 (y_3 - \bar{y})^2 = \frac{1}{12} \cdot 10 \cdot 120^3 + 1200 \cdot (75 - 70.83)^2$$

Summing these moments of inertia will give the total moment of inertia for the I-beam:

$$I = I_1 + I_2 + I_3$$

Let's calculate these values:

- For the top flange:

$$I_1 = \frac{1}{12} \cdot 50 \cdot 15^3 + 750 \cdot (127.5 - 70.83)^2 \approx \frac{1}{12} \cdot 50 \cdot 3375 + 750 \cdot 3237.17 = 14062.5 + 2427877.5 = 2441939.12 \text{ mm}^4$$

- For the bottom flange:

$$I_2 = \frac{1}{12} \cdot 50 \cdot 15^3 + 750 \cdot (70.83 - 7.5)^2 \approx \frac{1}{12} \cdot 50 \cdot 3375 + 750 \cdot 4007.38 = 14062.5 + 3005538.6 = 3019601.1 \text{ mm}^4$$

- For the web:

$$I_3 = \frac{1}{12} \cdot 10 \cdot 120^3 + 1200 \cdot (75 - 70.83)^2 \approx \frac{1}{12} \cdot 10 \cdot 1728000 + 1200 \cdot 17.36 = 1440000 + 20832 = 1460832 \text{ mm}^4$$

Finally, summing these moments of inertia:

$$I = 2441939.12 + 3019601.1 + 1460832 = 6922372.22 \text{ mm}^4$$

4. Determine the maximum bending stress:

The distance from the neutral axis to the outermost fiber c is:

$$c = 150 \text{ mm} - \bar{y} = 150 \text{ mm} - 70.83 \text{ mm} = 79.17 \text{ mm}$$

The internal moment M is given as 30 kNm, which is 30000 N·m or 30000000 N·mm.

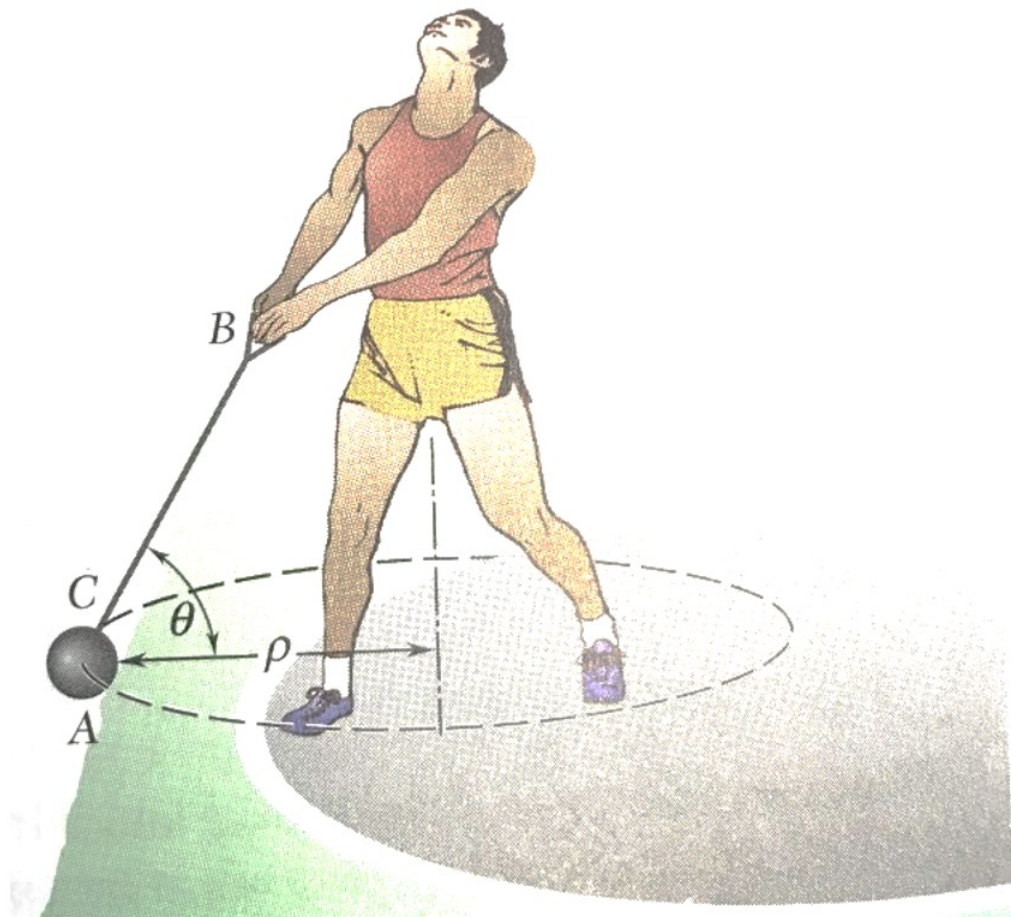
Using the flexure formula:

$$\sigma = \frac{Mc}{I} = \frac{30000000 \cdot 79.17}{6922372.22} \approx 343.04 \text{ MPa}$$

The maximum bending stress in the I-beam is approximately 343.04 MPa.

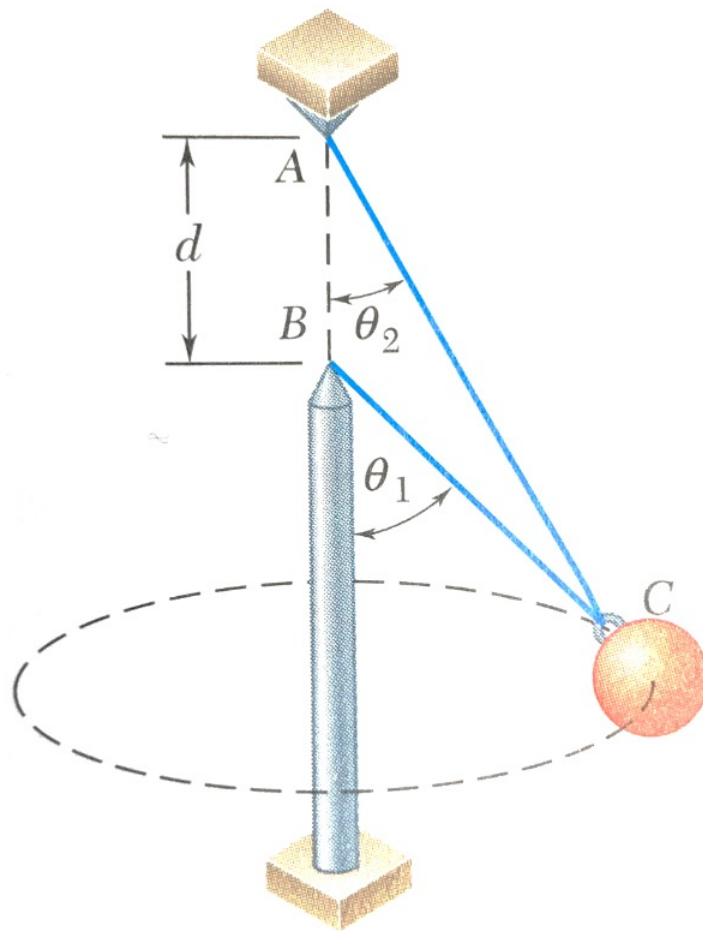
Problem Set 5 | Particle Dynamics Solve by hand or by Matlab, your choice

1. An athlete is practicing throwing a hammer. During the initial swings, the mass is revolving with a constant magnitude of the angular velocity ω in a horizontal circle as shown. Assuming that the head of the hammer can be modeled as a particle, determine the magnitude of the angular velocity vector $\vec{\omega}$, if $\theta = 60^\circ$ and $\rho = 1$ m.



Answer | $\omega = 2.38 \frac{\text{Rad}}{\text{s}}$

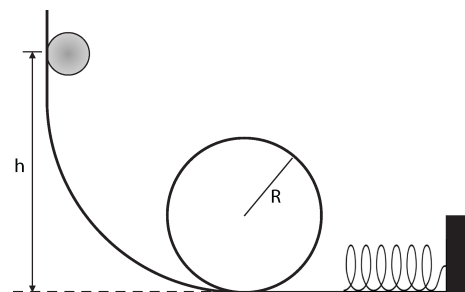
2. A single wire ABC passes through a ring C as shown in the image below. The ring C is attached to a mass of 5 kg that revolves in the horizontal circle, as shown. Assuming that $\theta_1 = 50^\circ$, $d = 0.8$ m, and the tension in both portion of the wire is 34 N, determine (a) the angle θ_2 , and (b) the velocity of the mass. You can choose whether the ball's rotation is clockwise or counterclockwise.



Answer | $\theta_2 = 36.9^\circ$ $v = 3.88 \frac{m}{s}$

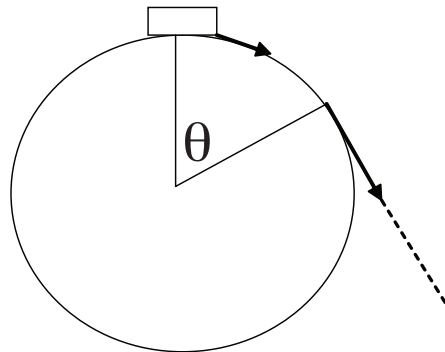
3. A marble of mass m initially at rest at height h moves along a frictionless rail that forms a hoop of radius R . At the end of the rail the marble hits a spring of elastic constant κ . Calculate:

- (a) the minimum value of h for which the marble hits the spring
- (b) the maximum compression of the spring for that value of h



Answer | (a) $h_{min} = \frac{5}{2}R$; (b) $\Delta x = \sqrt{\frac{5mgR}{\kappa}}$.

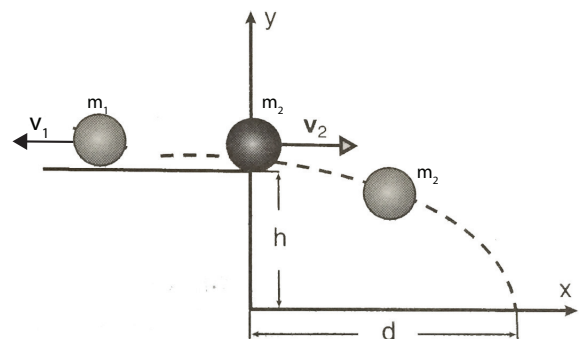
4. A small mass m starts from rest and slides from the top of a fixed sphere of radius r .



- (a) If the sphere is friction-less, at what angle θ from the vertical does the mass leave the surface?
- (b) Suppose there is friction between the mass and the sphere with friction coefficient $\mu_s=0.1$. What is the minimum angle θ_{min} at which the mass will start to slide along the sphere?
- (c) The mass is now placed just past this minimum angle and released. The coefficient of kinetic friction μ_k is small but not zero. Does the mass fly off at a larger or a smaller θ than found in part (a)? Assume θ is defined with respect to the top of the sphere.

Answer | (a) $\theta = 48.2^\circ$; (b) $\theta = 5.7^\circ$; (c) larger.

5. A marble of mass m_1 slides on a horizontal smooth surface with velocity v_0 until it collides elastically with a second marble of mass $m_2 = 3m_1$, which was initially at rest. The marble of mass m_2 falls from a step of height $h = 0.5m$. Find the value of v_0 for which the marble touches the ground a distance $d = 5m$ from the step.



Answer | 31.3 m/s.

6. A ball is dropped from the top of a 20 m building at the same time that another ball is thrown upward from a height of 2 m above the ground. If the two balls pass on another at the height of 10 m, determine the speed at which the second ball was thrown upward.

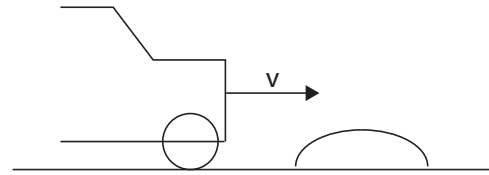
Answer | 12.6 m/s.

7. I have solved this using ChatGPT. Examine the AI solution and point out any errors. Reflect on the difference/similarities between your work and the AI

A 'sleeping policeman' is a bump placed in the

road to restrict the speed of cars. What radius of curvature should the bump have to cause cars traveling faster than 40 km/hr to leave the ground?

Answer | 12.6 m.





To determine the radius of curvature R of a bump (sleeping policeman) that will cause cars traveling faster than 40 km/h to leave the ground, we need to consider the forces acting on the car as it travels over the bump.

When a car travels over a bump, it experiences centripetal force. If this centripetal force exceeds the gravitational force, the car will lose contact with the ground and effectively "leave the ground."

The condition for the car to leave the ground is that the normal force becomes zero. At this point, the only force providing the centripetal acceleration is the weight of the car.

Let's denote:

- v as the speed of the car,
- g as the acceleration due to gravity (approximately 9.81 m/s^2),
- R as the radius of curvature of the bump.

The centripetal force required to keep the car on the curved path is given by:

$$F_{\text{centripetal}} = \frac{mv^2}{R}$$

The gravitational force acting on the car is:

$$F_{\text{gravity}} = mg$$

For the car to leave the ground, the centripetal force must equal the gravitational force:

$$\frac{mv^2}{R} = mg$$

We can cancel the mass m from both sides:

$$\frac{v^2}{R} = g$$

Solving for R :

$$R = \frac{v^2}{g}$$

Now, we need to convert the speed from km/h to m/s. Given that $1 \text{ km/h} = \frac{1}{3.6} \text{ m/s}$:

$$v = 40 \text{ km/h} = 40 \times \frac{1}{3.6} \text{ m/s} = \frac{40}{3.6} \text{ m/s} \approx 11.11 \text{ m/s}$$

Plugging the values into the equation for R :

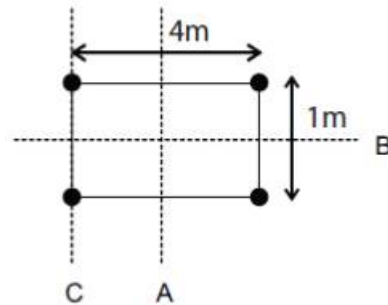
$$R = \frac{(11.11 \text{ m/s})^2}{9.81 \text{ m/s}^2} \approx \frac{123.21 \text{ m}^2/\text{s}^2}{9.81 \text{ m/s}^2} \approx 12.56 \text{ m}$$

So, the radius of curvature R of the bump should be approximately 12.56 meters for cars traveling faster than 40 km/h to leave the ground.

Homework Set 6.1: Your choice of by hand or Matlab

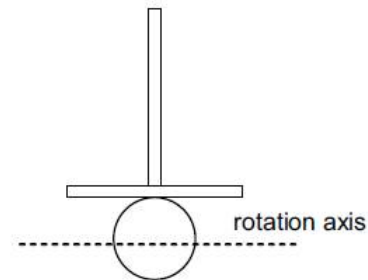
1. The rigid body in the picture shows four

10kg spheres connected by four light rods. Neglect the mass of the rods. a) Which is greater, the rotational inertia about axis A or the rotation inertia about axis B? b) What is the rotational inertia about axis C?



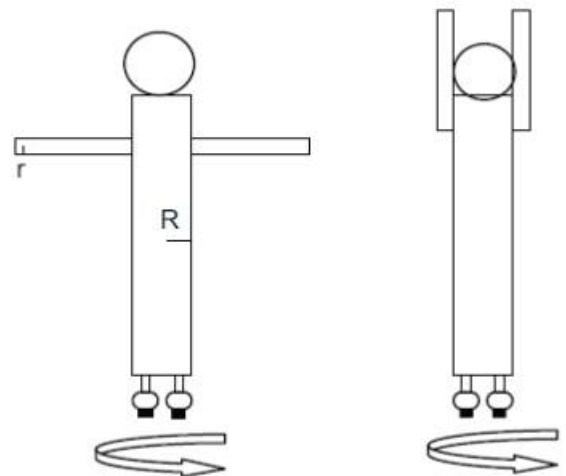
Answer | a) $I_A > I_B$, $I_A = 160kg\ m^2$, $I_B = 10kg\ m^2$ b) $I_C = 320kg\ m^2$

2. A rigid sculpture consists of a thin hoop of mass m and radius R and two thin rods each of mass m and length $L = 2R$. The sculpture can pivot around a horizontal axis in the plane of the hoop passing through its center. What is the sculpture's rotational inertia I about the rotation axis in terms of m and R .



Answer | $I = 5.8mR^2$

3. An ice skater can be modeled by a cylinder of radius R and length L (trunk and legs together), 2 small thin cylinders of radius r and length l (arms), and a sphere also of radius R (head). The head has mass m_h , the trunk m_t , and the arms m_a . The ice skater rotates about her central axis. Find the moment of inertia of the ice skater with arms outstretched horizontally and then with arms vertical.



$$I_{\text{Horizontal}} = \frac{2m_h R^2}{5} + \frac{m_t R^2}{2} + \frac{m_a (3r^2 + L^2)}{6} + 2m_a \left(\frac{L}{2} + R\right)^2$$

$$I_{\text{Vertical}} = \frac{2m_h R^2}{5} + \frac{m_t R^2}{2} + m_a r^2 + 2m_a (r + R)^2$$

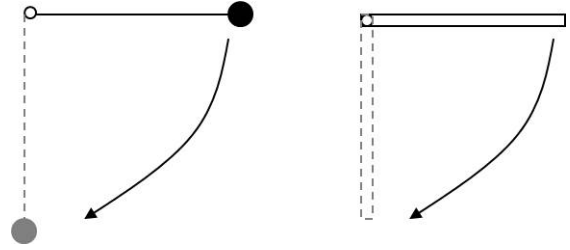
4. A simple pendulum hanging from a string of length L and a homogenous rod of length l that pivots at its end are released from the horizontal positions.

(a) What is the ratio of their angular velocities if their lengths are identical?

(b) How long does the rod have to be to have equal angular velocity?

$$I_{rod} = \frac{1}{12}mL^2$$

$$Answer \mid \frac{\omega_{pend}}{\omega_{rod}} = \sqrt{\frac{2}{3}}; l = \frac{3}{2}L.$$



5. I have solved this using hat T. xamine the AI solution and point out any errors. effect on the difference/similarities between your work and the AI

You crouch from a standing position, lowering your center of mass 18 cm in the process.

Then you jump vertically. The average force exerted on you by the floor while you jump is three times your weight. a) What is your upward speed as you pass through your standing position in leaving the floor? b) If all the energy is transferred into a flip with an angular velocity of 5 rad/s and the person's mass is 93kg, what is their Moment of Inertia?

Answer | a) 2.7 m/s b) I=39.4 kg*m^2



To solve this problem, we need to apply the principles of mechanics, specifically kinematics and dynamics.

Part (a): Upward speed as you pass through your standing position

1. Determine the acceleration:

- The average force exerted on you by the floor is three times your weight.
- Weight $W = mg$, where m is mass and g is the acceleration due to gravity (9.8 m/s^2).
- Given force $F = 3W = 3mg$.

2. Newton's Second Law:

- $F = ma$, where a is the acceleration.
- $3mg = ma \Rightarrow a = 3g$.

3. Energy considerations:

- The work done by the force while you crouch is converted into kinetic energy.
- Work done $W = F \cdot d$, where d is the distance over which the force acts ($18 \text{ cm} = 0.18 \text{ m}$).
- $W = (3mg) \cdot 0.18$.

4. Kinetic Energy:

- $W = \Delta K$, where ΔK is the change in kinetic energy.
- Initially, the kinetic energy $K_i = 0$.
- Final kinetic energy $K_f = \frac{1}{2}mv^2$.
- $(3mg) \cdot 0.18 = \frac{1}{2}mv^2$.
- Simplifying:

$$(3g) \cdot 0.18 = \frac{1}{2}v^2 \Rightarrow v^2 = 2 \cdot 3g \cdot 0.18 \Rightarrow v^2 = 1.08g \Rightarrow v = \sqrt{1.08g} \Rightarrow v = \sqrt{1.08 \cdot 9.8}$$

$$v \approx \sqrt{10.584} \approx 3.25 \text{ m/s}$$

Therefore, the upward speed as you pass through your standing position is approximately 3.25 m/s .

Part (b): Moment of Inertia if all energy is transferred into a flip

1. Angular Kinetic Energy:

- If all the kinetic energy is transferred into rotational kinetic energy, then:
- Rotational kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular velocity.
- Given $\omega = 5 \text{ rad/s}$.

2. Kinetic Energy from Part (a):

- Total kinetic energy $K = \frac{1}{2}mv^2$.
- Using $v = 3.25 \text{ m/s}$ and $m = 93 \text{ kg}$:

$$K = \frac{1}{2} \cdot 93 \cdot (3.25)^2 = \frac{1}{2} \cdot 93 \cdot 10.5625 = 491.15625 \text{ J}$$

3. Solving for Moment of Inertia:

- Set the rotational kinetic energy equal to the total kinetic energy:

$$\frac{1}{2}I\omega^2 = 491.15625 \text{ J} \Rightarrow I\omega^2 = 2 \cdot 491.15625 \Rightarrow I \cdot (5)^2 = 982.3125$$

$$I \cdot 25 = 982.3125 \Rightarrow I = \frac{982.3125}{25} \Rightarrow I = 39.2925 \text{ kg} \cdot \text{m}^2$$

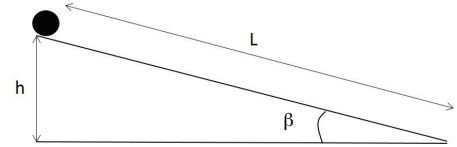
Therefore, the moment of inertia is approximately $39.29 \text{ kg} \cdot \text{m}^2$.

Problem Set 6.2 - Rigid Body Energy and Momentum

1. A ball with mass m and radius R is released from rest and rolls without slipping down a slope of length L that forms an angle β with the horizontal axis. The surface of the slope has coefficient of static friction μ_s . $I_{sphere} = \frac{2}{5}mR^2$.

(a) Calculate the velocity of the ball at the bottom of the ramp.

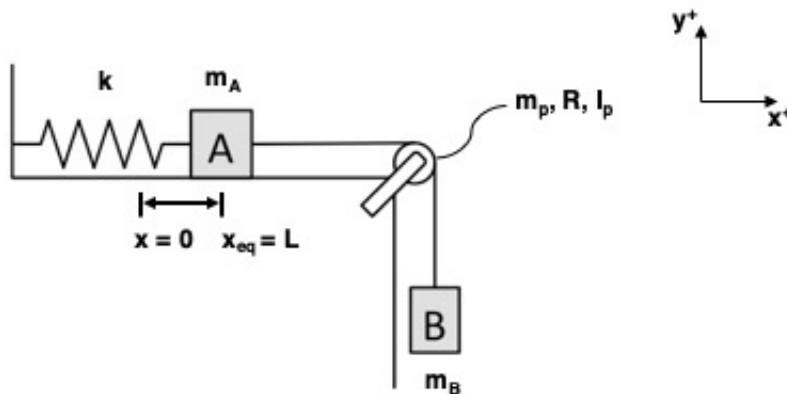
(b) Calculate the angle θ at which the ball starts to slide rather than roll down the slope.



Answer | a. $v_{CM/G} = \sqrt{\frac{10}{7}gL \sin \beta}$; b. $\tan \beta = \frac{7}{2}\mu_s$

2. Mass A ($m_A = 2.5\text{kg}$) lies on a smooth, frictionless surface attached to a spring of stiffness $k = 100\text{N/m}$ as shown. Mass B ($m_B = 1.5\text{kg}$) is connected to mass A by a light, inextensible string and is hanging below a pulley ($m_P = 2\text{kg}$, $R_P = 5\text{cm}$,

$I_{CM} = \frac{1}{2}mR^2$). Initially, the system is at equilibrium when the spring is extended by a distance $x = L = 15\text{cm}$.

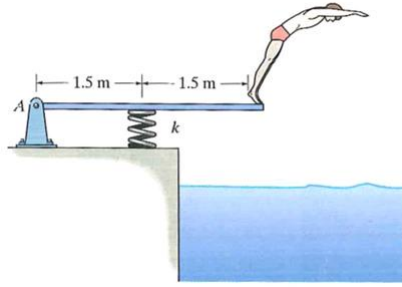


- (a) From the equilibrium position, mass B is then pulled downwards by a distance of 45cm and released. Find the speed of mass A (v_A) when it passes through the previous equilibrium point ($x = 15\text{cm}$). Let the acceleration due to gravity be equal to $g = 10\text{m/s}^2$.
- (b) Instead of a smooth, frictionless surface, let mass A lie on a surface with friction ($\mu_k = 0.1$). What is the speed of mass A when it passes through the equilibrium point ($x = 15\text{cm}$)? Note, this assumes the friction will only begin acting after mass B is pulled down 0.45m .

Answer (a) $v_A = -2.01\text{m/s}$; (b) $v_A = -1.90\text{m/s}$.

3. A diving board of length $L = 3m$ and mass $m = 30kg$ is supported by a spring of constant $\kappa = 5kN/m$. A diver stands on the edge of the board, causing the spring to compress $\Delta l = 300mm$. As the divers jumps off the board determine:

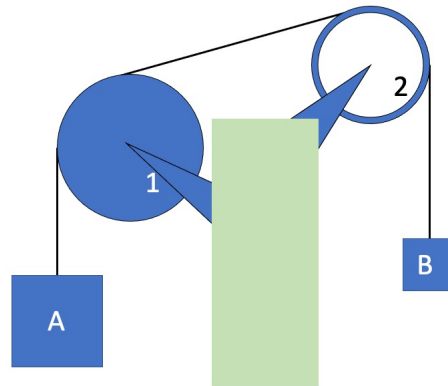
- (a) the angular acceleration of the board.
Assume the board is horizontal.
- (b) the horizontal and vertical components of the reaction at the pin A .



Answer | a. $\alpha = 20.1 \frac{Rad}{s^2}$ b. $R_t = 301.2N$, $R_n = 0N$.

4. An inextensible rope is strung over two pulleys, connecting mass A ($10.0kg$) to mass B ($6.00kg$). Pulley 1 is a solid thin disk, with a mass of $0.55kg$, and a radius of $0.12m$. Pulley 2 is a thin ring, has mass a $0.28kg$, and a radius of $0.08m$. The rope does not slip.

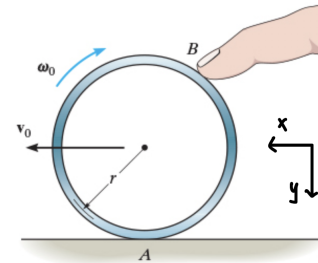
- (a) Determine the magnitude of the acceleration of mass A.
- (b) What is the magnitude and the direction of the angular acceleration of pulley number 2?



Answer | a. $a_A = 2.370 \frac{m}{s^2}$ b. $\alpha_2 = 29.63 \frac{Rad}{s^2}$ Counterclockwise

5. A thin ring of mass m and radius r is given an initial velocity v_0 and a backspin ω_0 . The coefficient of static and dynamic friction between the ring and the surface are μ_s and μ_k .

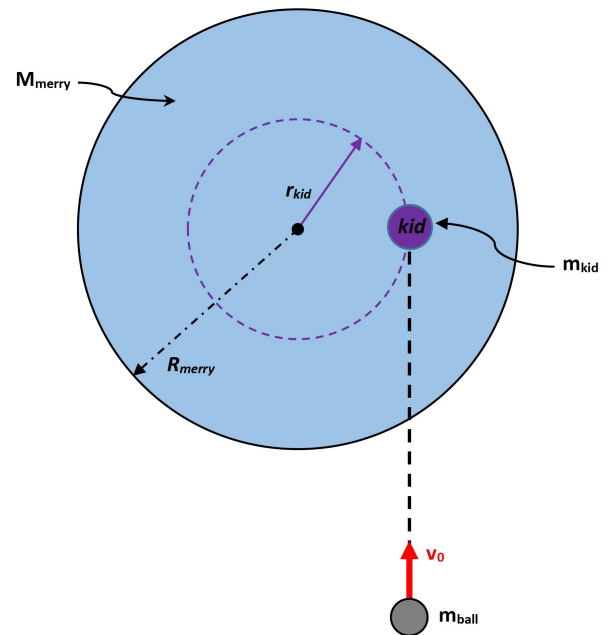
- (a) How far does the ring travel before the backspin stops?
- (b) What happens after? Why?



Answer | a. $d = -\frac{\omega_0 R}{\mu_k g} (\frac{1}{2} \omega_0 R + v_0)$ b. The rings rolls forward.

6. I have answered this questions using ChatGPT. Examine the AI answer, point out any errors with the solution. Reflect on how your work is similar/different than the AI

A kid of mass $m_k = 20kg$ stands on a merry-go-round at $r_k = 1m$ from the center. The merry-go-round has mass $M_{merry} = 30kg$ and radius $R_{merry} = 2m$. A ball is thrown to the kid with a velocity $v_b = 50m/s$ tangential to the merry-go-round as shown. The kid catches the ball. How heavy does the ball need to be so that the kid rotates all the way around the merry-go-round in 10 seconds? Model the merry-go-round as a disk ($I_{disk,CM} = \frac{1}{2}mR^2$), and the ball and kid as particles.





To solve this problem, we need to determine the mass of the ball that will provide enough angular momentum to the system so that the kid on the merry-go-round completes a full rotation in 10 seconds.

First, we can find the required angular velocity (ω) for the merry-go-round to complete one full revolution in 10 seconds.

1. Calculate the angular velocity:

$$\omega = \frac{2\pi}{T}$$

where T is the period (10 seconds).

$$\omega = \frac{2\pi}{10} = \frac{\pi}{5} \text{ rad/s}$$

2. Determine the moment of inertia of the system:

The moment of inertia of the merry-go-round (considered as a disk) is given by:

$$I_{\text{merry}} = \frac{1}{2} M_{\text{merry}} R_{\text{merry}}^2$$

Substituting the given values:

$$I_{\text{merry}} = \frac{1}{2} \times 30 \text{ kg} \times (2 \text{ m})^2 = \frac{1}{2} \times 30 \times 4 = 60 \text{ kg} \cdot \text{m}^2$$

The moment of inertia of the kid (considered as a particle) at a distance r_k from the center is:

$$I_{\text{kid}} = m_k r_k^2$$

Substituting the given values:

$$I_{\text{kid}} = 20 \text{ kg} \times (1 \text{ m})^2 = 20 \text{ kg} \cdot \text{m}^2$$

The total initial moment of inertia of the system (without the ball) is:

$$I_{\text{total}} = I_{\text{merry}} + I_{\text{kid}} = 60 + 20 = 80 \text{ kg} \cdot \text{m}^2$$

3. Calculate the required angular momentum:

The final angular momentum (L_f) needed to achieve the required angular velocity is:

$$L_f = I_{\text{total}} \omega = 80 \times \frac{\pi}{5} = 16\pi \text{ kg} \cdot \text{m}^2/\text{s}$$

4. Determine the angular momentum imparted by the ball:

When the ball (with mass m_b) is caught by the kid, its tangential velocity imparts angular momentum to the system. The angular momentum of the ball when caught by the kid is:

$$L_{\text{ball}} = m_b \cdot v_b \cdot r_k$$

Setting this equal to the required angular momentum:

$$L_f = L_{\text{ball}}$$

$$16\pi = m_b \cdot 50 \cdot 1$$

$$m_b = \frac{16\pi}{50}$$

$$m_b \approx 1.005 \text{ kg}$$

So, the ball needs to have a mass of approximately 1.005 kg for the kid to rotate all the way around the merry-go-round in 10 seconds.